2018 - 2019

Name of the lecturer : Dr. P. Gayathri
Group : III B.Sc.
Paper - V : Ring theory and Vector Calculus
VI : Laplace transforms

S.No	Name of the Student	P-V Asn-I 11.7.18	P-V Asn-II 3.10.18	Sem-V P-VI Asn-I 18.7.18	Sem-V P-VI Asn-II 3.10.18
1.	M. Venkatesh	Rings	Vector Differentiation	Laplace Transform-I	Application of L.T.
2.	Arabjan				
3.	K. Nitya Pooja Reddy				
4.	Nagaraja				
5.	Mahendra				
6.	Venkatramana				
7.	R. Praveen				
8.	Y. Rambabu				

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Name of the lecturer : Dr. P. Gayathri
Group : III B.Sc.
Paper : VII : Linear Algebra - I
Cluster Papers VIII A : Integral Transforms
VIII B : Numerical Analysis
VIII C - Linear Algebra - II

S.No.	Name of the Student	L.A-I A1 1/11	L.A-I A2 [illegible]	Integral Trans. A1 18/11	Integral Trans. A2 17/12	Numerical A1 18/11	Numerical A2 21/12	L.A-II A1 16/11	L.A-II A2 27/12
1.	M. Venkatesh	Vector spaces	Linear Transforms	Application of I.T	Fourier Transforms	Numerical Com. P.	Interpolation-I	Rank of a matrix	Inner Product Space
2.	Arabian								
3.	K. Nitya								
4.	Nagaraja								
5.	Mahendra								
6.	Venkatramana								
7.	R. Praveen								
8.	Y. Rambabu								

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Name of the lecturer : Dr. P. Gayathri

Group : II B.Sc.

Paper – II – Abstract Algebra and Real Analysis

S.No	Name of the Student	Sem-III Asn-I 5/7/18	Asn-II 1/10/18	Sem-IV Asn-I 8/1/19	Asn-[illegible]
1	M. Saleem	Groups	Normal Subgroups	Infinite Series	Riemann [illegible]
2.	S. Mohammad Ghouse				
3.	S. Masud Alam				
4.	B. Karthik				
5.	D. Venkateswarlu				
6	K. Annamaiah				
7.	S. Mohammad Ziya				
8.	D. Reddy Sekhar				

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Name of the lecturer : Dr. P. Gayathri

Group : II B.Sc.

Paper : Differential Equations and Solid geometry

S.No	Name of the Student	Sem-I Asn-I 16/7/18	Asn-II 1/10/18	Sem-II Asn-I 8/1/19	Asn-II 14/2/19
1.	B. V. Sivamma	Differential equations	Exact D.E	Planes	Cones
2.	C. Chennakesava				
3.	G. Hari Priya				
4.	K. Suresh				
5.	M. Venugopal				
6.	M. Raj Kumar				
7	R. Sri Hari				
8.	S. Md. Huzaifa				
9.	S. Shameemathaj				
10.	Syed Najma				

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2019-2020

Name of the lecturer: Dr. P. Gayathri
Group : III B.Sc.
Paper : V - Ring theory and Vector Calculus

Semester - V

S.No.	Name of the Student	Ass-I 25/7/19	Ass-II 19/9/19	Ass-III 17/10/19
1.	M. Saleem	Rings	Vector Differentiation	Vector Calculus
2.	S. Mohammad Ghouse			
3.	S. Masud Alam			
4.	B. Karthik			
5.	D. Venkateswarlu			
6.	K. Annamaiah			
7.	S. Mohammad Ziya			
8.	D. Reddy Sekhar			

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Name of the lecturer : Dr. P. Gayathri
Group : III B.Sc.
Paper : VI - Laplace Transforms

Semester - V

S.No.	Name of the Student	Ass-I 26/7/19	Ass-II 20/9/19	Ass-III 18/10/19
1	M. Saleem	Laplace Transformation	Inverse Laplace Transformation	Application of Laplace Transformation
2.	S. Mohammad Ghouse			
3.	S. Masud Alam			
4.	B. Karthik			
5.	D. Venkateswarlu			
6.	K. Annamaiah			
7.	S. Mohammad Ziya			
8.	D. Reddy Sekhar			

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Name of the Lecturer: Dr. P. Gayathri
Group : II B.Sc.
Paper - III - Abstract Algebra

Semester - III

S.No.	Name of the Student	Ass - I 27/7/19	Ass - II 23/9/19	Ass - III 14/10/19
1.	D. V. Sivamma	Groups	Normal Subgroups	Cyclic group
2.	C. Chennakesava			
3.	G. Hari priya			
4.	K. Suresh			
5.	M. Venugopal			
6.	M. Raj kumar			
7.	R. Sri Hari			
8.	S. Md. Huzaifa			
9.	S. Shameem Ahamad			
10.	Syed Najma			

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Name of the lecturer : Dr. P. Gayathri
Group : I B.Sc.
Paper - I : Differential Equations.

Semester - I

S.No.	Name of the Student	Ass - I 30/7/19	Ass - II 24/9/19	Ass - III 15/10/19
1.	K. V. Prasad	Differential Equations	Exact D. E.	Higher Order D. E.
2.	M. Lalitha			
3.	S. Md. Rafi			
4.	V. Mahesh			
5.	G. V. Rajesh			
6.	G. Supriya			
7.	D. Nagesh			
8.	G. Sai Hari			
9.	J. Rajasekhar			
10.	K. Prasad Naidu			
11.	V. S. Kalyan			
12.	S. Ashok			
13.	T. Druthi			
14.	V. V. Prasad			
15.	G. Apsa			
16.	S. Baraddin			
17.	S. Arhi			

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Name of the lecturer : Dr. P. Gayathri
Group : III B.Sc
Paper : VII - Linear Algebra

Semester - VI

S.No	Name of the Student	Ass-I 28.11.19	Ass-II 23.01.20	Ass-III
1.	M. Saleem	Vector Spaces	Dimension of a subspace	-
2.	S. Mohammad Ghouse			
3.	S. Masud Alam			
4.	B. Karthik			
5.	D. Venkateswarlu			
6.	K. Annamaiah			
7.	S. Mohammed Ziya			
8.	D. Reddy Sekhar			

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Name of the Lecturer : Dr. P. Gayathri
Group II B.Sc.
Paper IV - Real Analysis

Semester - IV

S.No.	Name of the Student	Ass-I 23/12/19	Ass-II 27/01/20	Ass-III
1.	B. V. Sivamma	Infinite Series	Limits and Continuity	-
2.	C. Chennakesava			
3.	G. Hari Priya			
4.	K. Suresh			
5.	M. Venu gopal			
6.	M. Raj Kumar			
7.	R. Sri Hari			
8.	S. Md. Huzaifa			
9.	S. Shameem Ahamed			
10.	Syed Najma			

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Name of the Lecturer: Dr. P. Gayatri
Group : I B.Sc.
Paper : II - Solid geom.

Semester - II

S.No.	Name of the Student	Ass-I 24/12/19	Ass-II 21/01/20	A[illegible]
1.	K.V. Prasad			
2.	M. Lalitha			
3.	S. Md. Rafi			
4.	V. Mahesh			
5.	G.V. Rajesh			
6.	G. Supriya			
7.	G. Sai Hari		Lines	
8.	J. Rajasekhar			
9.	K. Prasad Naidu	Plane		
10.	V.S. Kalyan			
11.	S. Ashok			
12.	T. Druthi			
13.	V.V. Prasad			
14.	G. Afsa			
15.	S. Bavaddin			
16.	S. Akhil			

12/8/20

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2020-2021

Name of the Lecturer: Dr. P. Gayathri
Group : III B.Sc.
Paper - II : Ring theory and Vector Calculus

Semester - V

S.No.	Name of the Student	Ass-I 20/11/2020	Ass-II [illegible]	Ass-III [illegible]
1	D.V. Sivamma			
2.	C. Chennakesava			
3.	G. Haripriya			
4.	K. Suresh			
5.	M. Venugopal			
6.	M. Raj Kumar	Rings	Vectors	Vector Calculus
7.	R. Sri Hari			
8.	S. Md. Huzaifa			
9.	S. Shameem Ahamed			
10				

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Name of the Lecturer: Dr. P. Gayathri
Group: III B.Sc
Paper: VI - Laplace Transf[illegible]

Semester - V

S.No	Name of the Student	Ass - I 2/4/2020	Ass - II 10/4/[illegible]	As[illegible]
1	D.V. Sivamma	Laplace Transformation	Inverse L.T.	Applications [illegible]
2.	C. Chennakesava			
3	G. Hari Priya			
4.	K. Suresh			
5	M. Venugopal			
6	M. Raj Kumar			
7.	R. Sri Hari			
8	S. Md. Huzaifa			
9.	S. Shameem Ahamad			

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Name of the Lecturer: Dr. P. Gayathri
Group: II B.Sc
Paper: III - Abstract Algebra

Semester - III

S.No.	Name of the Student	Ass - I 25/1/20[illegible]	Ass - II [illegible]	Ass - III [illegible]
1.	K.V. Prasad	Groups.	Normal Sub groups.	Cyclic groups
2	M. Lalitha			
3.	V. Mahesh			
4.	G.V. Rajesh			
5.	G. Supriya			
6.	G. Sai Hari			
7.	K. Prasad Naidu			
8.	V.S. Kalyan			
9	S. Ashok			
10	T. Druthi			
11	V.V. Prasad			
12.	G. Afsa			
13.	S. Bavoddin			
14.	S. Akhil			

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Name of the Lecturer: Dr. P. Gayathri
Group : III B.Sc.
Paper - VII : Linear Alge.

Semester - VI

S.No.	Name of the Student	Ass-I 14/6/24	Ass-II 17/7/24	Ass-III 26/...
1.	D.V. Sivamma	Vector spaces	Dimention of a subspaces.	vector space Isomorphism
2.	C. Chennakesava			
3.	G. Hari Priya			
4.	K. Suresh			
5.	M. Venugopal			
6.	M. Raj Kumar			
7.	P. Sri Hari			
8.	S. Md Huzaifa			
9.	S. Shameem Ahamed			

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Name of the Lecturer: Dr. P. Gayathri
Group : II B.Sc.
Paper - IV : Real Analysis

Semester - II

S.No.	Name of the Student	Ass-I 14/6/24	Ass-II 17/7/24	Ass-III 26/...
1.	K.V. Prasad	Infinite series	Limits & Continuity	Riemann Integration
2.	M. Lalitha			
3.	S. Md. Rafi			
4.	V. Mahesh			
5.	G.V. Rajesh			
6.	G. Supriya			
7.	G. Sai Hari			
8.	T. Rajasekhar			
9.	K. Prasad naidu			
10.	V.S. Kalyan			
11.	S. Ashok			
12.	T. Druthi			
13.	V.V. Prasad			
14.	G. Afia			
15.	S. Devaddy			
16.	S. Akhil			

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Name of the Lecturer : Dr. P. Gayathri
Group : I B.Sc.
Paper : Differential equa

Semester - I

S.No.	Name of the Student	Ass-I 16/3/21	Ass-II 23/4/21
1.	B. Harinadh		
2.	C. Thirumalesu		
3.	D. Prasanna		
4.	D. Pavankumar		
5.	G.V. Tharun		
6	K. Bhagya Rekha		
7.	K. Manjunadha		
8.	K. Bhanu Prakash		
9.	M. Sai	Ordinary D.E.	Higher order D.E.
10	M. Damodara		
11	P. Nanda Kumar		
12.	P. Satish Kumar		
13.	S. Sreedhar		
14.	S. Khader Basha		
15.	S. Md. Jabeed		
16	S. Suhel		
17.	S. Abdul		
18.	S. Mazeeda		
19.	S. Muneera		
20	T. Thirumalesh		
21.	V. Nageswara		
22	Venam Asha		
23	Y. Ganesh.		

GOVERNMENT DEGREE COLLEGE

RAYACHOTY – 516269, ANNAMAYYA DISTRICT. (A.P.)

DEPARTMENT OF Mathematics

(UG courses)

Assignment Topic

RANK OF A MATRIX

Topic Submitted
BY

Name of the Student : M. SAI

Class : II M.P.CS

Date : 16/07/2022

Academic Year: 2021 – 2022

GOVT. DEGREE COLLEGE: RAYACHOTY
DEPARTMENT OF Mathematics

ASSIGNMENT REGISTER

S.NO	DATE	CLASS	TOPIC
	16/7/2024	II MPCs	Rank of a Matrix

Sl. NO	Name of the Student	Signature of the Student
1	C. Tirumalesu	Tirumalesu
2	D. PRASANNA JYOTHI	Prasanna Jyothi
3	D. PAVAN KUMAR	Pavan Kumar
4	G. VENKATA TARUN	Venkata tarun
5	K. BHAGYAREKHA	Bhagyarekha
6	K. MANJUNADHA	Manjunadha.
7	K. BHANU PRAKASH	Bhanuprakash
8	M. SAI	Sai
9	M. DAMODARA	Damodara
10	P. NANDA KUMAR REDDY	Nandakumar reddy
11	P. SATISH KUMAR	Satish Kumar
12	S. SREEDHAR REDDY	sreedhar reddy
13	S. KHADER BASHA	S. Khader Basha
14	S. MOHAMMAD JABEED	S. Md. Jabeed
15	S. SUHEL	Suhel
16	S. ABDUL MAHAMMAD	Abdul Mahammad
17	S. MAZEEDA	Mazzeda
18	S. MUNEERA	Muneera
19	T. TIRUMALESH	T. Tirumalesu
20	V. NAGESWARA	Nageswara
21	V. ADI	Adi
22	Y. GANESH.	Y Ganesh.
23		
24		
25		
26		
27		
28		
29		
30		

Signature of the Lecturer

Signature of the Department I/C

Assignment - I

Reduce the matrix $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}_{4\times4}$ to normal form and hence find its rank.

Sol: Given matrix $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

$R_1 \leftrightarrow R_2 \sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1 \sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$

$R_4 \to R_4 - 6R_1$

$C_2 \to C_2 + C_1$

$C_3 \to C_3 + 2C_1 \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$

$C_4 \to C_4 + 4C_1$

$R_2 \to R_2 - R_3 \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -6 & -3 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$

$R_3 \to R_3 - 4R_2$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -6 & -3 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 66 & 44 \end{bmatrix}$

$R_4 \to R_4 - 9R_2$

$C_3 \to \frac{C_3}{33}$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 2 \end{bmatrix}$

$C_4 \to \frac{C_4}{22}$

$R_4 \to R_4 - 2R_3 \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix}$

$\therefore \rho(A) = 3$

2) Find the inverse of the matrix $A = \begin{bmatrix} 0 & 1 & 2 & 2 \\ 1 & 1 & 2 & 3 \\ 2 & 2 & 2 & 3 \\ 3 & 3 & 3 & 3 \end{bmatrix}$

Sol: Given that $A = \begin{bmatrix} 0 & 1 & 2 & 2 \\ 1 & 1 & 2 & 3 \\ 2 & 2 & 2 & 3 \\ 3 & 3 & 3 & 3 \end{bmatrix}_{4\times 4}$

Consider $A = I_4 A$

$$\Rightarrow \begin{bmatrix} 0 & 1 & 2 & 2 \\ 1 & 1 & 2 & 3 \\ 2 & 2 & 2 & 3 \\ 3 & 3 & 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A$$

$$R_1 \leftrightarrow R_2 \sim \begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 1 & 2 & 2 \\ 2 & 2 & 2 & 3 \\ 3 & 3 & 3 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A.$$

$$\begin{matrix} R_3 \to R_3 - 2R_1 \\ R_4 \to R_4 - 3R_1 \end{matrix} \sim \begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & -2 & -3 \\ 0 & 0 & -3 & -6 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & -3 & 0 & 1 \end{bmatrix} A$$

$$R_1 \to R_1 - R_2 \sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & -2 & -3 \\ 0 & 0 & -3 & -6 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & -3 & 0 & 1 \end{bmatrix} A$$

$$R_3 \to R_3 - R_4 \sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & -3 & -6 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & -3 & 0 & 1 \end{bmatrix} A$$

$$R_2 \to R_2 - 2R_3,\quad R_4 \to R_4 + 3R_3 \sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & -2 & -2 & -2 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 3 & -2 \end{bmatrix} A$$

$$R_4 \to \frac{R_4}{R_3} \sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & -2 & -2 & -2 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & -2/3 \end{bmatrix} A$$

$$R_1 \to R_1 - R_4,\quad R_2 \to R_2 + 4R_4,\quad R_3 \to R_3 - 3R_4 \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 & -1 & 2/3 \\ 1 & -2 & 2 & -2/3 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2/3 \end{bmatrix} A$$

$$\Rightarrow I_4 = B \cdot A \quad \text{where } B = \begin{bmatrix} -1 & 1 & -1 & 2/3 \\ 1 & -2 & 2 & -2/3 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2/3 \end{bmatrix}$$

$$\Rightarrow I_4 A^{-1} = (BA)A^{-1}$$

$$\Rightarrow A^{-1} = B(A \cdot A^{-1})$$

$$\Rightarrow A^{-1} = BI_4$$

$$\Rightarrow A^{-1} = B = \begin{bmatrix} -1 & 1 & -1 & 2/3 \\ 1 & -2 & 2 & -2/3 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2/3 \end{bmatrix} //$$

$\boxed{3}$ Solve $x+y-z+t=0,\ x-y+2z-t=0,\ 3x+y+t=0.$

Sol: Given system is $x+y-z+t=0,\ x-y+\cancel{2+2}2z-t=0,$
$3x+y+t=0$

This can be expressed as $\begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ i.e $AX=0$

we $A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix}$, $O = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Reducing to echelon form.

$R_2 - R_1$
$R_3 - 3R_1$ $\sim \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & -2 & 3 & -2 \\ 0 & -2 & 3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$R_3 - R_2$ $\sim \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & -2 & 3 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

This is in echelon form.

$\rho(A) = 2 <$ no. of variables in $X(4)$

$\therefore AX = 0$ has no zero solutions only.

$\Rightarrow x + y - z + t = 0 \rightarrow$ (i)

$\Rightarrow -2y + 3z - 2t = 0 \rightarrow$ (ii)

Let $z = k_1$; $t = k_2$ where k_1, k_2 are two parameters.

(ii) $\Rightarrow -2y + 3z - 2t = 0$

$\Rightarrow 2y = 3z - 2t$

$\Rightarrow y = \frac{3k_1 + 2k_2}{2}$

(i) $\Rightarrow x + y - z + t = 0$

$\Rightarrow x = -y + z - t$

$x = -\left[\frac{3k_1 - 2k_2}{2}\right] + k_1 - k_2$

$\therefore x = \frac{-3k_1 + 2k_2 + 2k_1 - 2k_2}{2} = \frac{-k_1}{2}$

$\therefore X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = k_1 \begin{bmatrix} -1/2 \\ 3/2 \\ 1 \end{bmatrix} + k_2 \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$

ie $x-y+2z+t=2,\ 3x+2y+t=1,\ 4x+y+2z+2t=3.$

Given equations $x-y+2z+t=2,\ 3x+2y+t=1,\ 4x+y+2z+2t=3$

The system can be expressed as

$$\begin{bmatrix} 1 & -1 & 2 & 1 \\ 3 & 2 & 0 & 1 \\ 4 & 1 & 2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \quad \text{i.e } AX = B$$

Reducing to echelon form

$$\begin{matrix} R_2 - 3R_1 \\ R_3 - 4R_1 \end{matrix} \sim \begin{bmatrix} 1 & -1 & 2 & 1 \\ 0 & 5 & -6 & -2 \\ 0 & 5 & -6 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ -5 \end{bmatrix}$$

$$R_3 - R_2 \sim \begin{bmatrix} 1 & -1 & 2 & 1 \\ 0 & 5 & -6 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ 0 \end{bmatrix}$$

This is in echelon form.

$\rho(A) = 2 = \rho(AB) < $ no. of variables in $X(4)$

$\therefore AX = B$ has infinity many solutions.

$\Rightarrow x - y + 2z + t = 2$ — (i)

$\Rightarrow 5y - 6z - 2t = -5$ — (ii)

Let $z = k_1,\ t = k_2$ where k_1, k_2 are two parameters

(ii) $\Rightarrow 5y - 6z - 2t = -5$

$\Rightarrow 5y - 6k_1 - 2k_2 = -5$

$\Rightarrow 5y = -5 + 6k_1 + 2k_2$

$\Rightarrow y = \dfrac{-5 + 6k_1 + 2k_2}{5}$

(i) $\Rightarrow x - y + 2z + t = 2$

$$\Rightarrow x = 2 + \left[\frac{-5+6k_1+2k_2}{5}\right] - 2k_1 - k_2$$

$$\Rightarrow x = \frac{10-5+6k_1+2k_2-10k_1-5k_2}{5}$$

$$\Rightarrow x = \frac{5-4k_1-3k_2}{5}$$

$$\therefore X = \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} +1 \\ -1 \\ 0 \\ 0 \end{bmatrix} + k_1 \begin{bmatrix} -4/5 \\ 6/5 \\ 1 \\ 0 \end{bmatrix} + k_2 \begin{bmatrix} -3/5 \\ 2/5 \\ 0 \\ 1 \end{bmatrix}$$

5) Find the characteristic roots and the corresponding characteristic vectors of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.

Sol: Given matrix is $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

The characteristic equation of A is $|A-\lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (8-\lambda)\left[(7-\lambda)(3-\lambda)-8\right] - (-6)\left[-6(3-\lambda)+8\right] + 2\left[24-2(7-\lambda)\right]$$

$$\Rightarrow (8-\lambda)(21-10\lambda+\lambda^2-16)+6(-18+6\lambda+8)+2(24-14+2\lambda)=0$$

$$\Rightarrow (8-\lambda)(\lambda^2-10\lambda+5)+6(6\lambda-10)+2(2\lambda+10)=0$$

$$\Rightarrow 8\lambda^2-80\lambda+40-\lambda^3+10\lambda^2-5\lambda+36\lambda-60+4\lambda+20=0$$

$$\Rightarrow -\lambda^3+18\lambda^2-45\lambda=0$$

$$\Rightarrow \lambda(-\lambda^2+18\lambda-45)=0$$

$$\Rightarrow \lambda(-\lambda^2+15\lambda+3\lambda-45)=0$$

$$\Rightarrow \lambda\left[\lambda(-\lambda+15)-3(-\lambda+15)\right]=0$$

$\lambda[(-\lambda+15)(\lambda-3)]=0$

$\Rightarrow \lambda = 0, 3, 15$

The characteristic roots of A are 0, 3, 15.

<u>Case (i)</u> :-

Let $\lambda=0$ characteristic vectors corresponding to the characteristic root '0' are given by $(A-0I)X=0$.

$$\Rightarrow \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_1 \leftrightarrow R_3 \sim \begin{bmatrix} 2 & -4 & 3 \\ -6 & 7 & -4 \\ 8 & -6 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{matrix} R_2+3R_1 \\ R_3-4R_1 \end{matrix} \sim \begin{bmatrix} 2 & -4 & 3 \\ 0 & -5 & 5 \\ 0 & 10 & -10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_3+2R_2 \sim \begin{bmatrix} 2 & -4 & 3 \\ 0 & -5 & 5 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow 2x-4y+3z=0$ (i)

$-5y+5z=0 \Rightarrow$ (ii).

Let $z=k$ $(\neq 0)$

(ii) $\Rightarrow -5y+5z=0$

$\Rightarrow 5y=5z$

$\Rightarrow y=z$

$\Rightarrow \boxed{y=k}$

(i) $\Rightarrow 2x-4y+3z=0$

$2x = 4k - 3k$

$\Rightarrow 2x = k$

$\Rightarrow \boxed{x = k/2}$

$\therefore$ Characteristic vectors corresponding to the characteristic root "0" are given by $X = k\begin{bmatrix} 1/2 \\ 1 \\ 1 \end{bmatrix}$ where k is non-zero parameters.

Case(ii): Let $\lambda = 3$ characteristic vectors corresponding the characteristic root 3 are given by $(A - 3I)X = 0$

$$\Rightarrow \left\{\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}\right\}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$\begin{matrix} 5R_2 + 6R_1 \\ 5R_3 - 2R_1 \end{matrix} \sim \begin{bmatrix} 5 & -6 & 2 \\ 0 & -16 & 8 \\ 0 & -8 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$2R_3 - R_2 \sim \begin{bmatrix} 5 & -6 & 2 \\ 0 & -16 & -8 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$\Rightarrow 5x - 6y + 2z = 0$ —(i)

$-16y - 8z = 0$ —(ii)

Let $z = k \ (\neq 0)$.

(ii) $\Rightarrow -16y - 8z = 0$

$\Rightarrow -16y = 8z$

$\Rightarrow -16y = 8k$

$\boxed{y = -\tfrac{1}{2}k}$

$5x - 6y + 2z = 0$

$\Rightarrow 5x = 6y - 2z$

$\Rightarrow 5x = 6\left(\frac{-k}{2}\right) - 2k$

$\Rightarrow 5x = -3k - 2k$

$\Rightarrow 5x = -5k$

$\Rightarrow \boxed{x = -k}$

$\therefore$ characteristic vectors corresponding to the characteristic root '3' are given by $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = k\begin{bmatrix} 1 \\ 1/2 \\ 1 \end{bmatrix}$ where k is non-zero parameters.

Case-(iii):- Let $\lambda = 15$

Characteristic vectors corresponding to the characteristic root 15 are given by $(A - 15I)X = 0$

$$\Rightarrow \left\{\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 15 \end{bmatrix}\right\} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -7 & -6 & 2 \\ -6 & -8 & -4 \\ 2 & -4 & -12 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{matrix} 7R_2 - 6R_1 \\ 7R_3 + 2R_1 \end{matrix} \sim \begin{bmatrix} -7 & -6 & 2 \\ 0 & -20 & -40 \\ 0 & -40 & -80 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow R_3 - 2R_2 \sim \begin{bmatrix} -7 & -6 & 2 \\ 0 & -20 & -40 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow -7x - 6y + 2z = 0$ —(i)

$-20y - 40z = 0$ i.e $y + 2z = 0$ —(ii)

Let $z = k$ $(\neq 0)$

(ii) $\Rightarrow y + 2z = 0 \Rightarrow y = -2z = -2k$ $\therefore y = -2k$

i) $\Rightarrow -7x - 6y + 2z = 0$ — (i)

$\cancel{-20y - 40z = 0 \quad \text{i.e. } y + 2z = 0}$ — (ii)

$\cancel{\text{Let } z = k \ (\neq 0)}$

(ii) $\Rightarrow 7x = -6y + 2z$

$\Rightarrow 7x = -6(-2k) + 2k = 14k$

$\boxed{\therefore x = 2k}$

$\therefore$ Eigen vectors corresponding to eigen root 15 are given by $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = k\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$ where k is non-zero parameter.

GOVERNMENT DEGREE COLLEGE

RAYACHOTY – 516269, ANNAMAYYA DISTRICT. (A.P.)

DEPARTMENT OF Mathematics

(UG courses)

Assignment Topic

Limits

Topic Submitted BY

Name of the Student : K. Likhitha

Class : II M.P.Cs

Date : 03-01-2023

Academic Year: 2022–2023

GOVT. DEGREE COLLEGE: RAYACHOTY

DEPARTMENT OF Mathematics

ASSIGNMENT REGISTER

S.NO	DATE	CLASS	TOPIC
	3/11/23	II MPCS	Limits

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3	T. Sreevidya	
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7	B. Sangeetha	
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B. M
Signature of the Lecturer

B. M
Signature of the Department I/C

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UNIT - 3

Limit's and continuity

Limit of a function:-

let $f: S \to R$ is a function 'a' be a limit point of aggregate 's' and $l \in R$

(i) The function 'f' tends to limit 'l' as 'x' tends to 'a' from left

(ii) If for each $\epsilon > 0, \exists \delta > 0$, s.t $x \in S$ and $a - \delta < x < a \Rightarrow |f(x) - l| < \epsilon$

(iii) we write $f(x) \to l$ as $x \to a^-$ (or) $\lim_{x \to a} f(x) = R$ (or)

$f(a-0) = l$ is called left hand limit of function. f tends to limit l as x tends to a from right

(i) for each $\epsilon > 0, \exists \delta > 0$ s.t $x \in S$ & $a < x < a + \delta \Rightarrow |f(x) - l| < \epsilon$ Right

(ii) we write $f(x) \to l$ as $x \to a^+$ (or) $\lim_{x \to a^+} f(x) = l$ (or) $f(a+0) = l$ is called right hand limit of the function.

→ The function f tends to limit l as x tends to a.

If for each $\epsilon > 0, \exists \delta > 0$ s.t $x \in S$ & $0 < |f(x) - l| < \epsilon$

→ we write $f(x) \to l$ as $x \to a$ (or) $\lim_{x \to a} f(x) = l$ is called limit of function

P. Define $f: S \to R$ such that $f(x) = x \sin(\frac{1}{x})$ using the definition of limit prove that $\lim_{x \to 0} f(x) = 0$.

clearly '0' is limit point of 's'

Here $f(x) = x \sin(\frac{1}{x})$

we have to show that $\lim_{x \to 0} f(x) = 0$

we must show that $\epsilon > 0 \ \exists \delta > 0$ s.t $|f(x) - 0| < \epsilon$ for $0 < |x - 0| < \delta, x \in S$

Now $|f(x) - 0| = |x \sin 1/x - 0| = |x \sin(\frac{1}{x})|$

$= |x| \cdot |\sin(1/x)|$ $\quad (\because \sin(\frac{1}{x}) \le 1)$

$\le x$

$\Rightarrow |f(x) - 0| < \epsilon$ where $0 < x < \epsilon$

choose $\delta = \epsilon$

we have $|f(x) - 0| < \epsilon$ where $0 < |x - 0| < \delta$ $x \in S$

Hence $\lim_{x \to 0} f(x) = 0$

i.e., $\lim_{x \to 0} x \sin(1/x) = 0$.

2) Define $f: R \to R$ s.t $f(x) = x^2+2x$ using the definition limit show that $\lim_{x\to 3} f(x) = 15$

Clearly '3' is limit point of R

Here $f(x) = x^2+2x$

we have show that $\lim_{x\to 3} f(x) = 15$

Now $|f(x)-15| \Rightarrow |x^2+2x-15| \Rightarrow |(x+5)(x-3)|$

$\Rightarrow |x+5|\,|x-3|$

If $|x-3| < 1$ then $2 < x < 4$ i.e., $x \in (2,4)$

$\Rightarrow x+5 \in (7,9)$

$\Rightarrow |x-3| < 1 \Rightarrow |x+5| < 9$

$\Rightarrow |x^2+2x-15| < 9\,|x-3|$

for $\epsilon > 0$. $9|x-3| < \epsilon \Leftrightarrow |x-3| < \frac{\epsilon}{9}$

If $\delta = \min\{1, \frac{\epsilon}{9}\}$ then

$\Rightarrow |x-3| < \delta \Rightarrow |x+2x-1| < 9, |x-3|$

for each $\epsilon > 0$, we take $\delta = \min\{1, \epsilon/9\}$

$\Rightarrow x \in R,\ 0 < |x-3| < \delta \Rightarrow |f(x)-15| < \epsilon$

$\lim_{x\to 3} f(x) = 15$

3) If $f(x) = \sin 1/x$, $x \in R - \{0\}$ prove that $\lim_{x\to 0} \sin 1/x$ doesn't exist

limit to not

If possible $\lim_{x\to 0} f(x) = l$

case (i) let $l \neq 1$

for $\epsilon = |l-1| > 0\ \exists\ \delta > 0$ s.t $0 < |x| < \delta \Rightarrow |\sin 1/x - l| < |l-1|$

By Archimedian property $\exists\ n \in N$ s.t $0 < \frac{1}{2n\pi + \frac{\pi}{2}} < \delta$

for $x = \frac{1}{2n\pi - \frac{\pi}{2}}$, $|\sin(2n\pi + \frac{\pi}{2}) - l| < |l-1|$

$\Rightarrow |1-l| < |l-1|$

This is impossible & hence $l \neq 1$ is not true

case (ii) let $l = 1$

for $\epsilon = 1\ \exists\ \delta > 0$ s.t $0 < |x| < \delta$

$|\sin(\frac{1}{x}) - 1| < 1$

for $0 < x = \frac{1}{n\pi} < \delta \Rightarrow |\sin n\pi - 1| < 1 \Rightarrow 1 < 1$

This is an impossible & hence $l = 1$ is not true

$\lim \sin 1/x$ doesn't exist

4) Prove that $\lim\limits_{x\to 0} \dfrac{3x+|x|}{7x-5|x|}$ doesn't exist

When $x<0$

$\Rightarrow \lim\limits_{x\to 0^-} \dfrac{3x+|x|}{7x-5|x|}$

$\Rightarrow \lim\limits_{x\to 0^-} \dfrac{2x}{12x} = 1/6$

When $x\geq 0$, $|x|=x$

$\Rightarrow \lim\limits_{x\to 0^+} \dfrac{3x+|x|}{7x-5|x|}$

$\lim\limits_{x\to 0^+} \dfrac{3x+x}{7x-5x}$

$\lim\limits_{x\to 0^+} \dfrac{4x}{2x} = 2$

$\lim\limits_{x\to 0} \dfrac{3x+|x|}{7x-5|x|}$ is doesn't exist

imp

When $x<0$, $|x|=-x$

$\Rightarrow \lim\limits_{x\to 0^-} \dfrac{3|x|-2x}{3x-2|x|}$

$\lim \dfrac{-5x}{5x}$

When $x\geq 0$, $|x|=x$

$\Rightarrow \lim\limits_{x\to 0^+} \dfrac{3|x|-2x}{3x-2|x|}$

$\lim\limits_{x\to 0^+} \dfrac{3x-2x}{3x-2x} = \dfrac{x}{x} = 1$

does not exist

5) If $f: R\to R$ is such $f(x)=[x]$ when $[x]$ denotes greatest integer not $>x$ then integral part of x show that $\lim\limits_{x\to 1} f(x)$

imp

$\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-} [x]$

$\Rightarrow \lim\limits_{h\to 0} [1-h]$

$\Rightarrow \lim\limits_{h\to 0} (0) = 0 \quad [\because 0\leq 1-h<1]$

$\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+} [x]$

$\lim\limits_{h\to 1^+} [1+h] = \lim\limits_{h\to 0} (1) = 1$

$\lim\limits_{x\to 0} f(x) \neq \lim\limits_{x\to 1^+} f(x)$

Hence $\lim\limits_{x\to 1} f(x)$ doesn't exist

Uniform contunity:-

let 'S' be an aggregate and $f: S \to R$ be a func. then 'f' is uniformly continuous on 'S' if given $\epsilon > 0$ $\exists$ $\delta > 0$ s.t. $x_1, x_2 \in S$, $|x_1 - x_2| < \delta \Rightarrow |f(x_1) - f(x_2)| < \epsilon$

Theorem:-

If a function f is continuous on closed interval a,b then it uniformly continuous on [a,b]

imp

Proof:- let $\epsilon > 0$

f is continuous on [a,b] $\Rightarrow$ for $\epsilon > 0$, we can divide [a,b] into finite number say 'n', of sub intervals.

$$[a = t_0, t_1], [t_1, t_2] \cdots [t_{r-1}, t_r][t_r, t_{r+1}] \cdots [t_{n-1}, t_n = b]$$

such that $|f(x_1) - f(x_2)| < \frac{\epsilon}{2} \longrightarrow ①$

for x_1, x_2 belonging to the same subinterval

let $\delta = \frac{1}{2} \min \{ |t_r - t_{r-1}| > 0, 1 \leq r \leq n \}$

let x_1, x_2 any two Points of [a,b] s.t $|x_1 - x_2| < \delta$ then x_1, x_2 either belong to the same subinterval or two consecutive subintervals with a common end point.

case (i):-

let x_1, x_2 belong to the same subinterval

$\Rightarrow |f(x_1) - f(x_2)| < \frac{\epsilon}{2} < \epsilon$ for $|x_1 - x_2| < \delta$ ($\because$ from ①)

case (ii):-

let x_1, x_2 belong to two consecutive sub-intervals with a common end point say t_r.

we have from eq ① $|f(x_1) - f(x_2)| < \frac{\epsilon}{2}$ and $|f(t_r) - f(x_2)| < \frac{\epsilon}{2}$

$\therefore |f(x_1) - f(x_2)| = |\{f(x_1) - f(t_r)\} + \{f(t_r) - f(x_2)\}|$

$\leq |f(x_1) - f(t_r)| + |f(t_r) - f(x_2)|$